The Present Key Importance of the Fine Structure Constant, α, to a Better Knowledge of All the Fundamental Physical Constants *

Jesse W. M. DuMond

California Institute of Technology, Pasadena, California

Dedicated to Prof. J. Mattauch on his 70th birthday

The dilemma is described which exists at the present time between the two present best sources of information as to the numerical value of the Sommerfeld fine structure constant, α. These two sources are the fine structure splitting in deuterium, determined in 1953 by Triebwasser, Dayhoff and Lamb, and the hyperfine structure splitting in hydrogen, measured more recently using the Ramsey hydrogen maser. The theoretical connection between the *fine structure* measurements and α is subject to little question but the experimental difficulties to obtain a precision of a few ppm are considerable. The relative precision obtained with the hydrogen maser on the other hand, is phenomenal (of order 10^{-11}) but the theoretical connection between the *hyperfine* splitting and α is subject to a controversial correction for the internal field structure of the proton. Assuming this correction term to be correct at its present value, the hf splitting in H implies a value of α 26 ppm higher than the fs splitting in D. Present existing sources of evidence, some favourable to the lower and some to the higher value of α, are presented and discussed and the key importance of a better knowledge of this fundamental constant is stressed.

1. The Dilemma Regarding α

Our two present best sources of information regarding the numerical value of the fine structure constant, α, of Sommerfeld give discordant values. The measurement [1] by Lamb, and associates of the fine structure splitting in deuterium gave $\alpha = (7.29720 \pm 0.00003) \times 10^{-3}$, whereas the highly precise measurement [2] of the hyperfine splitting in hydrogen by Ramsey and associates, which, however, requires the use of a controversial and uncertain theoretical factor for the proton internal field structure [3], gives $\alpha = 7.29739 \pm 0.00008$, a value 26 ± 12 ppm higher than the former [4]. In the Lamb fs measurements, a careful study (both theoretical and experimental) of the resonance line profile shape was performed in order to attain the final accuracy claimed, since this last was of the order of only one one-thousandth of the line width at half maximum. The possibility of a small systematic experimental error of a few parts per million cannot, therefore, be entirely ruled out. In the case of the hfs measurements in H, the work of the Ramsey group is probably one of the most precise measurements ever made by man, exceeding in accuracy the fs in D measurements by better than five orders of magnitude. The value of these measurements *as a source of information on* α, however, is clouded by the serious *theoretical* uncertainty regarding the validity of the present theory behind the correction for the proton's internal field structure. Not enough is known at present, either theoretically or experimentally, about the interaction at close range between polarized protons and polarized electrons to compute the correction with requisite certainty. In forming the 1963 least-squares adjustment of the fundamental constants [4], the structure of the equations of observation representing the data was such as to furnish no reliable criteria to decide between these two discrepant values of α by any test of over-all consistency. The choice in favor of the fs in D value was made because the theoretical uncertainty from the proton field-structure factor, needed to compute α from the hfs in H result, seemed to pose the larger question-mark.

* Work in part supported by a personal grant from National Science Foundation to J. W. M. DuMond.

[1] S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., Phys. Rev. 89, 98 [1953].

[2] D. Kleppner, H. M. Goldenberg, and N. F. Ramsey, Appl. Optics 1, 55 [1962]. — F. M. Pipkin and R. H. Lambert, Phys. Rev. 127, 787 [1962]. — S. B. Crampton, D. Kleppner, and N. F. Ramsey, Phys. Rev. Letters 11, 338 [1963]. — E. N. Fortson, D. Kleppner, and N. F. Ramsey, Phys. Rev. Letters 13, 22 [1964].

[3] C. K. Iddings and P. M. Platzman, Phys. Rev. 113, 192 [1959]. — D. E. Zwanziger, Bull. Amer. Phys. Soc. II, 6, 514 [1961], Abstr. Kl. — A. J. Layzer, Bull. Amer. Phys. Soc. II, 6, 514, Abstr. K 2.

[4] E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 [1965] ; also Proc. Second Internat. Conf. on Nuclidic Masses 1963, p. 152. Editor Walter Johnson, Springer-Verlag, Vienna 1964.

In performing analyses to determine "best" values of the fundamental constants at any given epoch, such problems of choice as this one typify the analyst's most taxing dilemma. Two distinct sets of values of the fundamental constants, each based on one of the two horns of the dilemma, would be likely to lead to confusion. To wait for more satisfactory criteria from further experiments would have meant indefinite delay. We were being urgently pressed, at the requests of two official bodies, one of national, the other of international scope, to issue a set of values promptly.

2. Determinations of the hfs Splitting in Muonium by Hughes and his Group

Because of this dilemma, HUGHES and his associates have undertaken determinations of the hyperfine splitting in muonium [5]. These results, which suffer from no such uncertain theoretical correction as is needed in the case of the hfs in hydrogen, yielded a value of α in accord with that inferred from the LAMB (et al.) − fs in D measurements. However, the present muonium result leaves something to be desired as a clinching verification in favor of the LAMB value of α because of certain corrections (by extrapolation) for the pressure of the gas in which the muonium was formed. The work with muonium by HUGHES and his associates was kindly undertaken by them because of our urgent need in 1962 and '63 for a criterion to choose between the two values of α. The speed and skill with which so difficult a measurement was obtained was highly commendable. Given more time and improved facilities for the work, it is to be hoped that this method can be improved to a point which will remove the last vestige of uncertainty regarding interpretation of the result.

3. Robiscoe's Work on the Level Differences, L and ΔE − L, in H and D

Because of the above mentioned small residual uncertainties surrounding the 1950 − '53 measurements of LAMB et al. on his famous shift, L, and on

the fs in D, ROBISCOE [6] has, very recently, at the University of Chicago, remeasured the level difference,

$$L = 2^2S_{1/2} - 2^2P_{1/2} \qquad (1)$$

for hydrogen by an independent method which he calls the crossing point method. Fig. 1 is a diagram
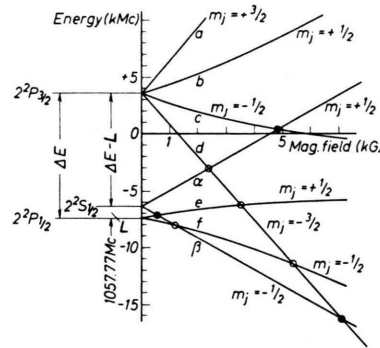

Fig. 1. [Taken from ROBISCOE [6]] ZEEMAN diagram of the fine structure in H, $n = 2$. The levels a, ..., β are LAMB's designation. The present experiment [Ref. [6]] is concerned with the measurement of the crossing point between levels β and e, near 575 G. The a−c and β−d crossing points could be measured by the present technique. The famous LAMB shift, L, and the level differences ΔE and ΔE−L have been added to the labelling of ROBISCOE's figure.

of the levels as a function of applied magnetic field intensity. ROBISCOE's results yielded a value of L significantly higher than LAMB's by about 0.3 ± 0.14 Mc sec^{-1}. ROBISCOE is now continuing these important measurements at Yale University and in a letter to me dated October 28, 1965, he reports his preliminary (as yet unpublished) results on (1) the shift, L, for deuterium, and on (2) the difference ΔE − L for hydrogen, where ΔE means the entire fine structure splitting, $2^2P_{3/2} - 2^2P_{1/2}$. ROBISCOE's value of

$$L_R = 1059.34 \pm 0.17 \text{ Mc sec}^{-1} \text{ (deuterium)} \quad (2)$$

for deuterium differs from LAMB's value for deuterium

$$L_L = 1059.00 \pm 0.10 \text{ Mc sec}^{-1} \text{ (deuterium)} \quad (3)$$

by the same systematic difference of 0.3 Mc sec^{-1} as he obtained in his results at Chicago for the case of hydrogen.

[5] V. W. HUGHES, R. PREPOST, J. BAILEY, and W. CLELAND, Phys. Rev. Letters **8**, 103 [1962]. − K. O. H. ZIOCK, V. W. HUGHES, R. PREPOST, J. M. BAILEY, and W. E. CELAND, Phys. Rev. Letters **8**, 103 [1962]. − R. PREPOST, J. M. BAILEY, W. E. CLELAND, M. ECKHAUSE, and V. W. HUGHES, Bull. Amer. Phys. Soc. **9**, 81 [1964]. − J. M. BAILEY and V. W.

HUGHES, Proc. Second Internat. Conf. on Nuclidic Masses 1963, p. 130, Editor WALTER JOHNSON, Springer-Verlag, Vienna 1964.

[6] R. T. ROBISCOE, Phys. Rev. **138**, A 22 [1965]. Also Doctoral Thesis, Univ. Chicago 1964.

ROBISCOE with two others, WILLIAMS and COSENS, has made two preliminary r.f. measurements of $\Delta E - L$ in hydrogen using distinctly different branchings of the level diagram which, nevertheless, agree very satisfactorily as to the result. He describes these as follows

$$\beta(m_f = -1) \to d(m_f = -2),$$
$$\nu = 8828 \text{ Mc}, \ Hc \sim 749 G, \quad (4)$$
$$\Delta E - L = 9911.0 \pm 0.5 \text{ Mc sec}^{-1},$$

$$\beta(m_f = -1) \to b(m_f = 0),$$
$$\nu = 11475 \text{ Mc}, \ Hc \sim 682 \text{ G}, \quad (5)$$
$$\Delta E - L = 9911.0 \pm 0.7 \text{ Mc sec}^{-1}.$$

Although LAMB never directly measured $\Delta E - L$ in hydrogen, it is easy to infer a value from his measurements of the same quantity for deuterium by means of the equation

$$(\Delta E - L)_H = (R_H/R_D) \Delta E_D - L_H \quad (6)$$

wherein H = hydrogen, D = deuterium and R is the RYDBERG. ROBISCOE's present preliminary results can thus be compared with LAMB's as follows:

LAMB JR. et al.

$$(\Delta E - L)_L = 9910.84 \pm 0.10 \text{ Mc sec}^{-1}, \quad (7)$$

ROBISCOE et al.

$$(\Delta E - L)_R = 9911.0 \pm 0.4 \text{ Mc sec}^{-1}. \quad (8)$$

The apparent discrepancy, namely something like $+0.2 \pm 0.4$ Mc sec^{-1}, is as yet too uncertain to be more than merely suggestive. It is hoped that continuation of the work with improved conditions will permit more satisfactorily definite conclusions. The rf field distribution is believed to exhibit too sharply varying a character transverse to the beam in the rf transition region at present and an improved instrumental design will be required. However, ROBISCOE is willing to speculate that an eventual difference between his (ROBISCOE's) value of ΔE_H from the value of that quantity to be inferred from LAMB's ΔE_D might conceivably be found to be as large as 0.5 Mc sec^{-1}, the ROBISCOE value being the higher. However, the present evidence is sufficiently uncertain so that ROBISCOE prefers to express the discrepancy (if such it be) as $+0.5 \pm 0.5$ Mc sec^{-1}. The inference is that α^2 might have to be increased by 50 ppm relative to the value used in our 1963 adjustment.

4. Indirect Evidence from x-ray and Gamma-Ray Spectroscopy Suggesting Need for Increase in α and Λ

I wish now to describe certain recently obtained indirect evidence from the field of precision x-ray and gamma-ray spectroscopy which points in the same direction as ROBISCOE's fs measurements, favoring the *higher* value of α and, indeed, suggesting that the correct value may equal or possibly even exceed the value quoted above for the result of the RAMSEY et al. hf splitting in hydrogen (with the present problematical proton structure correction). The evidence I shall present consists of ten examples of five different kinds of experiment, each here to be regarded as direct or indirect determinations of the conversion constant, $\Lambda = \lambda_g/\lambda_s$; the constant by which x-ray wavelengths expressed on the "x-unit" nominal scale must be multiplied to obtain the corresponding wavelength value in milliangstrom units. For the sake of definiteness, I shall here define the x-unit nominal scale as such that the wavelength at peak intensity of the Mo, $K\alpha_1$, line equals exactly 707.831 x-units.

(1) Two independent measurements made by KNOWLES at the Chalk River reactor, using his highly precise two-crystal spectrometer, to measure the wavelength of the annihilation radiation. In the first of these the annihilation occurred in water, and in the second in metallic tantalum [7]. The annihilation wavelength, $h/(mc)$, measured in absolute units may equally well be expressed as $\alpha^2/(2 R_\infty)$. If, as in KNOWLES' case, it is measured by crystal diffraction on the x-unit scale *, the result gives $\alpha^2/(2 R_\infty \Lambda)$, thus establishing a valuable relationship between α and Λ. The disagreement between the two KNOWLES measurements is a yet unexplained.

(2) A measurement by SIEGBAHN and associates in which the quantum-energy difference between two x-ray lines, Mo K α_1, and Cu K α_1, was directly

[7] J. W. KNOWLES, Canad. J. Phys. **37**, 203 [1959]; **40**, 237, 257 [1961]; Proc. Second Internat. Conf. on Nuclidic Masses 1963, p. 113, Editor WALTER JOHNSON, Springer-Verlag, Vienna 1964.

* Throughout this paper we adopt as an arbitrary definition of the term "x-unit" the wavelength of the Mo Kα_1 line at peak intensity divided by 707.831. In his recently prepared tables "X-Ray Wavelengths" [10], BEARDEN has used this scale essentially, taking however as his primary definition of 1 x.u. the wavelength λ (W Kα_1, at peak intensity)/208.5770 and establishing by measurement the secondary reference line, λ (Mo Kα_1, at peak intensity) = 707.831 x. u.

determined by measuring a retarding (or accelerating) potential applied to the photoelectrons ejected from targets by one or other of the two lines. This potential is adjusted so that both photoelectrons are made to have identical kinetic energy as indicated by the fact that they focus in identical orbits in the SIEGBAHN–SVARTHOLM-type double focusing magnetic beta-ray spectrometer. Given the x-ray wavelengths of the lines on the relative, x-unit, scale as determined by crystal diffraction measurements this work permits computation of the voltage-to-wavelength conversion constant, $h\,c^2/(e\,\Lambda)$, where Λ is again the x-unit-to-milliangstrom-unit conversion constant [8]. SIEGBAHN and his co-authors state that they believe the accuracy of the method can be greatly improved over that attained in this preliminary measurement. Because of its importance for our knowledge of the fundamental constants of Physics and Chemistry, they should be warmly encouraged to refine the measurement.

(3) A measurement of the short wavelenght limit of the continuous x-ray spectrum by SPIJKERMAN and BEARDEN using an x-ray tube with a target consisting of a jet of mercury vapor [9]. Like the work of SIEGBAHN et al. just referred to, this measurement leads also to a value of $h\,c^2/(e\,\Lambda)$.

(4) Numerous high precision measurements by BEARDEN [10] and associates including also collected earlier results of others, such as those of SMAKULA [11] on crystals of different species in which the macroscopic density of the crystal samples and the absolute volumes (in cubic x-units) of their unit cells are accurately determined, the latter by precision x-ray diffraction methods. If the crystal approaches ideal perfection as to chemical and structural purity, the quotient of the molecular weight of the unit cell of the crystal divided by the product of its density and unit cell volume in cubic x-units should give the combination, $N\,\Lambda^3$, where N is the AVOGADRO number.

In his recently published tables [10] entitled "X-Ray Wavelengths", BEARDEN has made an attempt to normalize his tabulated X-Ray wavelength values as nearly as possible in terms of angstrom units. The wavelength unit he employs, which he calls A* is *defined* by him, however, as such that the peak intensity point of the WK α_1 lines has the value

$$\lambda(\text{WK }\alpha_1,\text{ peak}) = 0.2090100 \text{ A*} . \qquad (9)$$

He states that this unit, A*, is equal to 1 angstrom ± 5 ppm. This, however, is a conclusion reached by BEARDEN *based on experimental measurements* on crystals of different species in which the measured quantities are (a) the macroscopic crystal density, ϱ, in gram cm^{-3}, (b) the volume, v, of the crystal unit-cell in cubic x-units, $v = d_x^3\,\Phi$, determined by x-ray diffraction **, and (c) the aggregate atomic or molecular weight, M_u, of the atoms comprising the unit cell. Such measurements actually, however, determine the product, $N\,\Lambda^3$, where N is the AVOGADRO number and Λ the x-unit to milliangstrom unit conversion factor, through the relation

$$N\,\Lambda^3 = \frac{M_u}{\varrho\,d_x^3\,\Phi} \cdot 10^{33} . \qquad (10)$$

In order to determine Λ from measurements of this sort one must therefore know the AVOGADRO number, N. For this purpose BEARDEN adopted the value of N published as the result of the COHEN-DuMOND 1963 least-squares adjustment, a value which in no wise depended on x-ray data of any sort but which, as we shall see, does depend strongly on the value of α used as one of the highly important input data in that adjustment.

(5) Direct determinations of Λ by diffraction at grazing incidence of x-ray emission lines from ruled gratings. These are of two types, the first by BEARDEN [10] using plane ruled gratings in which the absolute angles of incidence and diffraction were measured, and the second, originally done in Sweden by TYRÉN, using concave ruled gratings, in which the wavelengths of highly ionized, one-electron, hydrogenic-atom spark lines are compared on one and the same spectral plate with the wavelengths, in higher

[8] S. HAGSTRÖM, O. HORNFELDT, C. NORDLING, and K. SIEGBAHN, Ark. Fysik **23** (15), 145 [1962].

[9] J. J. SPIJKERMANN and J. A. BEARDEN, Phys. Rev. **134**, A 871 [1964].

[10] "X-Ray Wavelengths", J. A. BEARDEN, Final Report Contract AT (30-1)-2543 U.S. Atomic Energy Commission, Div. Tech. Info. Extension, Oak Ridge, Tenn. 1964. Contains all pertinent prior references. Values (3) through (8) in Figs. 2 through 4 of the present paper have been taken over from Table A of "X-Ray Wavelengths" without modification.

[11] A. SMAKULA and V. SILS, Phys. Rev. **99**, 1744 [1955]. — A. SMAKULA, J. KALNAJS, and V. SILS, Phys. Rev. **99**, 1747 [1955].

** The quantity, Φ, is a dimensionless factor, called the volume factor, by which d_x^3 must be multiplied to obtain the volume of the unit cell when the axes describing the latter (as in the case of calcite for example) are oblique. The angular measurements to determine Φ are readily made by x-ray diffraction methods.

orders of diffraction, of soft x-ray emission lines [12]. In 1956 we pointed out that in the 1940 work of TYRÉN the calculations of spark-line wavelengths had not been corrected for the then-unknown LAMB shift [13]. Accordingly, TYRÉN's plates have recently been remeasured by EDLÉN and SVENSSON and the data completely recalculated [14]. Their result yields a value, in absolute (angstrom) units, for the mean wavelength of the Al K $\alpha_1 \alpha_2$ doublet, 8.33992 ± 0.00020 Å. In order to deduce a value of the x-unit-to-milliangstrom-unit conversion constant, Λ, from this result, it is necessary to establish the weighted mean wavelength of the very soft Al K α_{12} x-ray doublet on our nominal x-unit scale. NORDFORS [15] and CAUCHOIS [16] are in disagreement by 70 ppm as to this wavelength. For this I have assumed a 2 : 1 intensity ratio of α_1 to α_2 and have used the new tabular values from the 1964 BEARDEN tables, Al K α_1 = 8322.063 and Al K α_2 = 8.324457 giving a weighted mean value of 8.322861 x-units (the x-unit being, as defined by BEARDEN, a unit such that λ (WK α_1 peak) = 208.5770 x-units or λ (MoK α_1, peak = 707.831 x-units). I conclude then that the EDLÉN–SVENSSON work implies a value

$$\Lambda = 1.002050 \pm 0.000024 . \qquad (11)$$

In our 1963 least-squares adjustment of the fundamental constants [4], as the result of careful study of a large list of items of data considered as possible candidates for use, and after making an analysis of variance [17] comprising the equivalent of about 150 L.-S. adjustments of different sub-sets of the data, we finally retained only the input data here listed in Table 1. No rejections were made of any datum on the *sole* basis of its being significantly outlying from the consensus, however. Each was studied carefully for internal weaknesses which might explain its possible affliction with systematic error. In the 1963 adjustment, after much study of all the x-ray data, and particularly the data of the above type (4), then available on $N \Lambda^3$, using the

Fine Structure in Deuterium [1]	10971.59 ± 0.10 Mc sec^{-1}
Magnetic Moment of the Proton in Nuclear Magnetons [18]	2.792757 ± 0.000025
Magnetic Moment of the Proton in Nuclear Magnetons [19]	2.792770 ± 0.000070
Faraday Constant by Electrolysis of Silver [20]	96486.82 ± 0.66 coul mol^{-1}
Gyromagnetic Ratio of the Proton [21]	26751.92 ± 0.08 sec^{-1}gauss^{-1}
Gyromagnetic Ratio of the Proton [22]	26751.88 ± 0.08 sec^{-1}gauss^{-1}

Table 1. Input data used in 1963 adjustment.

then-accepted tabular x-ray emission line wavelengths, we found so much inconsistency in this field, with no sure criteria for selection or rejection, that we felt forced to reject from that adjustment all x-ray data in toto [4, 17]. What has happened is simply that the gradually improving precision of data from other fields than x-rays has surpassed the precision then available in that field.

I have urged for the last 8 years certain very much needed conventional reforms in precision x-ray spectroscopy such as (a) the redefinition of the x-unit scale in terms of the wavelength of a selected standard x-ray emission line rather than in terms of the grating spacing of a *species* of crystal (Calcite) and, (b) the adoption of a well-defined feature (the peak) of the natural profile of an x-ray line as the fiducial wavelength to be quoted in tabulations. Calcite can vary in grating constant many tens of parts per million from sample to sample. This source of confusion and the lack of any well-defined operational convention as to what feature of the natural line profile is referred to when wavelengths are quoted constitute two sources responsible for much of the chaotic inconsistency that has long existed among x-ray spectroscopic data.

[12] F. TYRÉN, Z. Physik **109**, 722 [1938] ; Thesis, unpublished. Uppsala 1940.

[13] J. W. M. DuMOND and E. R. COHEN, Phys. Rev. **103**, 1583 [1956]. — J. W. M. DuMOND, Proc. Nat. Acad. Sci. **45**, 1052 [1959].

[14] B. EDLÉN and L. A. SVENSSON, Ark. Fysik **28** (36), 427 [1965].

[15] B. NORDFORS, Ark. Fysik **10** (21), 279 [1956].

[16] Y. CAUCHOIS, C. R. Acad. Sci. Paris **221**, 25 [1945].

[17] E. R. COHEN and J. W. M. DuMOND, Report to the National Research Council Committee on Fundamental Constants of Physics and Chemistry (Chairman, A.G. McNISH, Chief, Div. Metrology, U.S.-N.B.S.) "An Analysis of Variance of

the Data Available in 1962", Feb. 28, 1963 (Unpublished). See also E. R. COHEN and J. W. M. DuMOND, Rev. Mod. Phys. **37**, 537 [1965].

[18] H. SOMMER, H. A. THOMAS, and J. A. HIPPLE, Phys. Rev. **82**, 697 [1951].

[19] J. H. SANDERS and K. C. TURBERFIELD, Proc. Roy. Soc. London A **272**, 79 [1962].

[20] D. N. CRAIG, J. I. HOFFMAN, C. A. LAW, and W. J. HAMER, J. Res., Nat. Bur. Stands. **64 A**, 381 [1960].

[21] P. L. BENDER and R. L. DRISCOLL, Trans. I.R.E. Instr. I-7, 176 [1958].

[22] P. VIGOUREUX, Proc. Roy. Soc. London A **270**, 72 [1962].

Now, at last, in the new BEARDEN tables, "X-Ray Wavelengths", an effort has been made to observe these reforms so as to express all wavelengths on a consistent basis, defined operationally with as much precision as possible. A. misfortune, however, concerning BEARDEN's decision in his tables to express all of the tabulated wavelength values in terms of a unit, A*, which aims to be an approximation to 1 angstrom, is that if, as seems now increasingly likely, it is found necessary to revise the values of the fundamental constants of our COHEN–DUMOND 1963 L.S. adjustment (because of the need to revise α), the value of N used by BEARDEN to determine Λ, from his and others' measurements of $N\Lambda^3$, may prove to require modification downward by a relative change of the order of 3 times the relative increase required in α. This will imply a relative increase in Λ (above that used by BEARDEN in computing his "X-Ray Wavelengths") of the same number of ppm as whatever relative increase in α turns out to be required. Thus his 2700 tabulated wavelengths expressed in A* units, instead of being approximations to angstrom units to within ±5 ppm, could need a systematic upward correction throughout of order +26 ppm.

The six input items of the COHEN–DUMOND 1963 least-squares adjustment given in Table 1. are clearly of only four kinds. Listed in the same order as in the table, the resulting types of observational equation can be written in the following simple form in which the C's are numerics depending only on the physical measurement in question and on auxiliary constants. (By the latter we mean constants such as the velocity of light, c; the RYDBERG constant for infinite mass, R_∞ and others, all of which are so much more accurately known that their uncertainties contribute negligibly to the uncertainty of each particular numeric, C.)

$$\alpha = C(\alpha), \tag{12}$$

$$N\,e^2/\alpha^3 = C(\mu_p/\mu_n), \tag{13}$$

$$N\,e = C(F), \tag{14}$$

$$\alpha^3/e = C(\gamma_p). \tag{15}$$

While the data of Table 1 furnish us with six equations in the three unknowns, α, e, and N, [two of type (13) and two of type (15)] the above set of four types of equations is seen to be quite degenerate in the following senses, (a) that equation (12) is disjoint from the rest, and (b) that the remaining three form a homogeneous set insufficient to solve for any of the unknowns without the use of (12). The pivotal importance of (12), the fine structure constant, is thus emphasized.

In Fig. 2 we plot the values of the x-unit-to-milli-angstrom-unit conversion factor, Λ, as obtained from
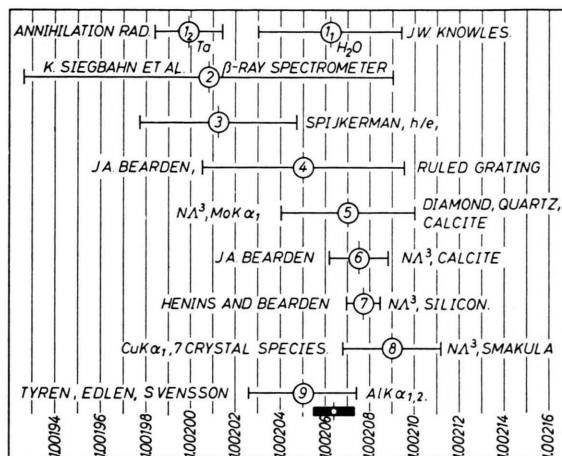

Fig. 2. Values of the x-unit-to-milliangstrom-unit conversion factor, Λ, calculated from ten diverse sources of information described in the text in Section 4, paragraphs 1 to 5 inclusive. References to these sources will be found listed in Table 2. In calculating the values of Λ numbered 1_1, 1_2, 2, 3, 5, 6, 7 and 8, the constants used were those from the 1963 COHEN–DUMOND least-squares adjustment predicated on the value $\alpha = (7.29720 \pm 0.00003)\,10^{-3}$ which was adopted from LAMB's et al's measurement of the fs splitting in D. The values of Λ numbered 4 and 9 are direct measurements, independent of other constants. The black bar at the bottom gives the weighted mean value of Λ and its standard deviation.

the diverse sources of information listed above under paragraphs (1) to (5) inclusive. These values of Λ, save for the ruled grating determinations (4), (9), and (10), have all required use of values of the fundamental constants for their calculation, and in Fig. 2 these values are taken directly from our 1963 adjustment. It will be clear from inspection that there are significant discrepancies between some of the different results on Λ.

Provided no new input data become available and no changes in the data of our 1963 adjustment are required other than a change in the numerical value of α, it is easy to verify from inspection of equations (12) to (15) above that any small relative increase in the value of α, which we may introduce arbitrarily, will result in relative increases twice as large in those values of Λ of Fig. 2 corresponding to the measurements of KNOWLES, of SIEGBAHN, and of SPIJKERMAN. The relative increase in the value of Λ derived from the several determinations of $N\Lambda^3$,

Item No., j	x_j (ppm)	$\sigma_j \cdot 10^6$	Weight w_j	$w_j x_j$	$w(x - \langle x \rangle)^2$	Source of Datum	
1_1	62	± 32.0	0.98	60.80	3.33	KNOWLES, λ_A in H_2O	a
1_2	-1	± 15.0	4.44	-4.44	18600.00	KNOWLES, λ_A in Ta	b
2	8	± 82.0	0.15	1.20	466.00	SIEGBAHN et al., Mag. Spectr.	c
3	12	± 35.0	0.82	9.84	2210.00	SPIJKERMAN et al., X.-S.W.L.	d
4	50	± 45.0	0.50	24.80	95.00	BEARDEN, Ruled grating	e
5	70	± 30.0	1.11	77.60	42.10	$N \Lambda^3$, Mo, Diamond, SiO_2, $CaCO_3$	f
6	75	± 13.0	5.90	443.00	735.00	BEARDEN, $N \Lambda^3$, $CaCO_3$	g
7	77	± 7.4	18.20	1400.00	3142.00	HENINS et al., $N \Lambda^3$, Si	f
8	90	± 22.0	2.06	185.50	1410.00	SMAKULA et al., $N \Lambda^3$, $CuK\alpha_{1,2}$	h
9	50	± 24.0	1.74	87.00	246.00	EDLÈN-SVENSSON-TYRÈN	i
			35.90	2285.30	26949.43		

$\langle x \rangle = \Sigma w_j x_j / \Sigma w_j = 64$, $\sigma_i = \pm 5.3$ ppm

$\Sigma w_j (x_j - \langle x \rangle)^2 = 26949.43$, $\sigma_e = \sqrt{26949.43/9(35.90)} = \pm 9.15$ ppm

$\langle \Lambda \rangle = 1.002064 \begin{cases} \pm 9.2 \text{ ext.} \\ \pm 5.3 \text{ int.;} \end{cases}$ $\chi^2 = 27.2$

a J. W. KNOWLES, Canad. J. Phys. **37**, 203 [1959]; **40**, 237, 257 [1961].
b J. W. KNOWLES, Proc. 2nd Internat. Conf. Nuclidic Masses 1963, p. 113. Ed. W. JOHNSON, Springer-Verlag, Vienna 1964.
c S. HAGSTRÖM, O. HORNFELDT, C. NORDLING, and K. SIEGBAHN, Ark. Fys. **23** (15), 145 [1962].
d J. J. SPIJKERMAN and J. A. BEARDEN, Phys. Rev. **134**, A 871 [1964].
e J. A. BEARDEN, Phys. Rev. **37**, 1210 [1931].
f I. HENINS and J. A. BEARDEN, Phys. Rev. **135**, A 890 [1964].
g J. A. BEARDEN, Phys. Rev. **137**, B 181 [1965].
h A. SMAKULA and V. SILS, Phys. Rev. **99**, 1744, 1747 [1955].
i B. EDLÉN and L. A. SVENSSON, Ark. Fys. **28** (36), 427 [1965].

Table 2. Weighted mean value of Λ if $\alpha = 7.29720 \cdot 10^{-3}$ (LAMB value). Let $x_j = (\Lambda_j - 1.002) 10^6$. $\sigma_j =$ Standard deviation of x_j; $w_j =$ weight. $\sigma_i =$ S. D. of mean by internal consistency. $\sigma_e =$ S. D. of mean by external consistency.

(5) to (8) inclusive, will, on the other hand, be *just equal* to whatever arbitrary relative increase we make in α. This is evident, since as can readily be verified from Eqs. (12) to (15), an upward relative change in α results in a downward relative change 3 times as large in N. Finally, the values of Λ numbered (3), and (9), derived from ruled grating diffraction experiments, are direct determinations which do not involve any constants from the 1963 adjustment and hence these remain fixed independent of any change we may make in α.

The weighted mean value of Λ corresponding to the values on this graph is $\Lambda_1 = 1.002064$ with a standard deviation by external consistency of 9.2 ppm and by internal consistency of 5.3 ppm. Thus the BIRGE ratio, $\sigma_e/\sigma_i = 1.74$, and $\chi^2 = 27.2$. There are 9 degrees of freedom. The details of this calculation are given in Table 2. In this calculation and the two succeeding it the standard deviations of the constants taken from the 1963 adjustment are treated as negligible.

Fig. 3 illustrates how the graph is modified when α is increased by 26 ppm relative to the LAMB et al., value used in our 1963 adjustment. This corresponds to the value required by the Harvard measurement of the hfs splitting in hydrogen using the IDDINGS-PLATZMAN-LAYZER-ZWANZIGER (controversial) cor-
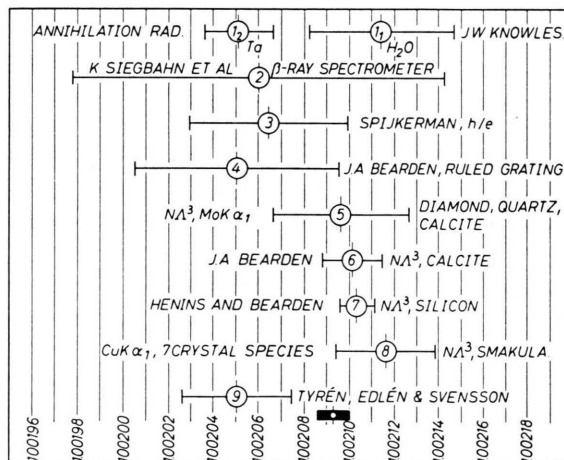

Fig. 3. Values of the x-unit-to-milliangstrom-unit conversion factor, Λ, calculated from the same sources of information as in Fig. 2 save that in this instance the constants used are predicated on a value of $\alpha = (7.29739 \pm 0.00003) 10^{-3}$, a value 26 ppm larger than the LAMB et al. value adopted in our 1963 L. S. adjustment. This larger value is the one computed from the hf splitting in hydrogen as measured by RAMSEY et al. with the hydrogen maser, assuming the IDDINGS-PLATZMAN-ZWANZIGER-LAYZER correction for proton internal field structure to be correct. The values of Λ numbered 4 and 9 are independent of other constants and remain unchanged. Relative to the corresponding items in Fig. 2, the value 1_1, 1_2, 2 and 3 are increased $2 \times 26 = 52$ ppm and the values 5 through 8 are increased 26 ppm because of the increase in α. The black bar at the bottom gives the weighted mean value of Λ and its standard deviation for this case.

Item No. j.	x_j (ppm)	$\sigma_j \cdot 10^6$	Weight w_j	$w_j x_j$	$w(x - \langle x \rangle)^2$	Source of Datum	
1_1	114	± 32.0	0.98	111.72	446.0	KNOWLES, λ_A in H_2O	a
1_2	51	± 15.0	4.44	226.44	7700.0	KNOWLES, λ_A in Ta	b
2	60	± 82.0	0.15	9.00	160.5	SIEGBAHN et al., Mag. Spectr.	c
3	64	± 35.0	0.82	52.48	676.0	SPIJKERMAN et al., X.-R. S. W. L.	d
4	50	± 45.0	0.50	24.80	905.0	BEARDEN, Ruled grating	e
5	96	± 30.0	1.11	106.56	12.1	$N\Lambda^3$. Mo, Diamond, SiO, $CaCO_3$	f
6	101	± 13.0	5.90	595.90	509.0	BEARDEN, $N\Lambda^3$, $CaCO_3$	g
7	103	± 7.4	18.20	1874.60	1930.0	HENINS et al., $N\Lambda^3$, Si	f
8	116	± 22.0	2.06	238.96	1120.0	SMAKULA et al., $N\Lambda^3$, $CuK\alpha_{1,2}$	h
9	50	± 24.0	1.74	87.00	3180.0	EDLÈN-SVENSSON-TYRÈN	i
			35.90	3327.46	16638.60		

$\langle x \rangle = \Sigma w_j x_j / \Sigma w_j = 92.70$,
$\Sigma w_j (x_j - \langle x \rangle)^2 = 16638.60$,
$\langle \Lambda \rangle = 1.002093 \begin{cases} \pm 7.17 \text{ ext.} \\ \pm 5.3 \text{ int.} \end{cases}$

$\sigma_i = 5.3$ ppm
$\sigma_e = \sqrt{16638.60/9(35.90)} = \pm 7.17$ ppm
$\chi^2 = 16.47$

Table 3. Weighted mean value of Λ if $\alpha = 7.29739 \cdot 10^{-3}$ (LAMB value $+26$ ppm). Let $x_j = (\Lambda_j - 1.002) 10^6$; $\sigma_j =$ Standard deviation of x_j; $w_j =$ weight. $\sigma_i =$ S. D. of mean by internal consistency. $\sigma_e =$ S. D. of mean by external consistency. References see Table 2.

rection term for proton internal field structure. With this upward modification in α by 26 ppm one obtains a weighted mean value of $\Lambda_2 = 1.002093$ with an S.D. of 7.17 ppm by external and 5.3 ppm by inter-
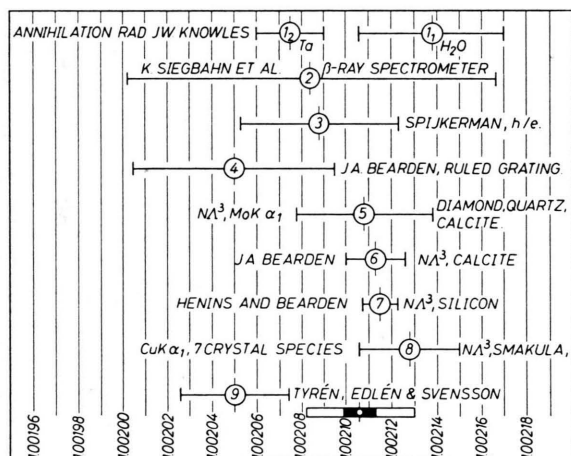

Fig. 4. Values of the x-unit-to-milliangstrom-unit conversion factor, Λ, calculated from the same sources of information as in Fig. 2 save that in this instance the constants used are predicated on a value of $\alpha = (7.29748 \pm 0.00003) 10^{-3}$, a value 38 ppm larger than the LAMB et al. value adopted in our 1963 L. S. adjustment. This value was selected as the result of a least-squares adjustment in two unknowns to determine that increment in α which would render the ten different data on Λ most consistent. The value $+38$ ppm from this adjustment had a standard deviation by external consistency $\sigma_e = \pm 24.7$ ppm. The weighted mean value of Λ for this case is shown at the bottom of the graph. The black bar shows the standard deviation by external consistency, if the value of α is regarded as *exact*, while the open rectangle shows the external standard deviation which reflects the standard deviation of ± 24.7 ppm in α.

nal consistency. Here the BIRGE ratio $\sigma_e / \sigma_i = 1.35$ and $\chi^2 = 16.47$. See Table 3 for details.

It is natural now to ask what relative change in α will result in the highest consistency for the graph, i. e., will minimize χ^2. It turns out that this is achieved by an increase in α of 38 ppm relative to our 1963 adopted value taken from the LAMB et al. F.S.-in-D work. Fig. 4 is the graph corresponding to this case and the value of Λ which results is

$$\Lambda_3 = 1.002106 .$$

Here the S.D. by external consistency is 6.9 ppm and by internal consistency, 5.3 ppm with $\sigma_e / \sigma_i = 1.30$ and $\chi^2 = 15.6$. For 9 degrees of freedom FISHER's tables indicate only about a 10 percent chance of obtaining a χ^2 as large or larger than this by accident, Table 4 gives the calculation for this case, assuming no uncertainty as to the increased value of α. In Fig. 4 the two standard deviations for the weighted mean value of Λ are plotted; the smaller assumes no uncertainty in the value of α while the larger reflects the uncertainty of the L.S. calculation.

Clearly no clinching degree of significance can be attached to these results. Like the results of ROBISCOE, they should be regarded as *suggestive*, rather than definitive. Undoubtedly, systematic errors *must* afflict some of the obviously internally inconsistent data on Λ in these graphs, such as the two values 1_1 and 1_2, of KNOWLES. All the direct ruled grating data are necessarily of low accuracy and correspondingly likely to be subject to systematic error.

Item No. j.	x_j (ppm)	$\sigma_j \cdot 10^6$	Weight w_j	$w_j x_j$	$w(x - \langle x \rangle)^2$	Source of Datum	
1_1	138	32.0	0.98	135.24	1000.00	Knowles, λ_A in H_2O	a
1_2	75	15.0	4.44	333.00	4300.00	Knowles, λ_A in Ta	b
2	84	82.0	0.15	12.60	73.50	Siegbahn et al., Mag. Spectr.	c
3	88	35.0	0.82	72.16	269.00	Spijkerman et al., X.-R.S.W.L.	d
4	50	45.0	0.50	24.75	1560.00	Bearden, Ruled grating	e
5	108	30.0	1.11	119.88	4.05	$N\Lambda^3$. Mo, Diamond, SiO_2, $CaCO_3$	f
6	113	13.0	5.90	666.70	283.00	Bearden, $N\Lambda^3$, $CaCO_3$	g
7	115	7.4	18.20	2093.00	1450.00	Henins et al., $N\Lambda^3$, Si	f
8	128	22.0	2,06	263.68	988.00	Smakula et al., $N\Lambda^3$, $CuK\alpha_{1.2}$	h
9	50	24.0	1.74	87.00	5400.00	Edlèn-Svensson-Tyrèn	i
			35.90	3808.01	15327.55		

$\langle x \rangle \Sigma w_j x_j / \Sigma w_j = 106.09,$ $\qquad \sigma_i = 5.3$ ppm

$\Sigma w_j (x_j - \langle x \rangle)^2 = 15327.55,$ $\qquad \sigma_e = \sqrt{15327.55/9\,(35.90)} = 6.88$ ppm

$\langle \Lambda \rangle = 1.002106 \begin{cases} \pm\ 0.0000069 \text{ ext.} \\ \pm\ 0.0000053 \text{ int.} \end{cases}$ $\qquad \chi^2 = 15.6$

Table 4. Weighted mean value of Λ if $\alpha = 7.29748 \cdot 10^{-3}$ (Lamb value $+38$ ppm). Let $x_j = (\Lambda_j - 1.002)\, 10^6$; $\sigma_j =$ Standard deviation of x_j; $w_j =$ weight. $\sigma_i =$ S. D. of mean by internal consistency; $\sigma_e =$ S. D. of mean by external consistency. References see Table 2. In Tables 2, 3 and 4, the σ_e-(S. D.)'s attached to the three weighted mean values of Λ are each computed as though the numerical value of α, in each of the 3 cases, were *an exactly known number*. This has been done for purposes of comparison of the r_e/r_i and χ^2 (measures of external compared to internal consistency) in the three cases. In Table 4 the value of $\alpha = 7.29748 = \alpha$ (Lamb $+ 38$ ppm) was selected as that value which would minimize χ^2, this selection being the result of a least-squares adjustment in two unknowns, x, and $y = [\alpha - \alpha(\text{Lamb})]/\alpha$. In this adjustment the increment above $\alpha(\text{Lamb})$, 38 ppm, had an uncertainty $\sigma_e = \pm 24.7$ ppm, however, and the weighted mean value of Λ, in order to correctly reflect *this* uncertainty should be $\langle \Lambda \rangle = 1.002106 \pm 0.000024$.

We feel, however, that there is at least qualitative significance in the fact that the data which are more sensitive to a change in α, of types 1 through 3, do indeed become more consistent with the less-α-sensitive $N\Lambda^3$-data, 5 through 8, when α is increased to a value roughly comparable with that required by the H.F.S. in H splitting (with the present admittedly ill-understood corrections for proton structure). Of particular interest is the improved consistency

Values as of 1963 L.S. Adjustment	New Values ** if α must be increased by 26 ppm.
$\alpha = (7.29720 \pm 0.00003) \times \times 10^{-3}$	7.29739×10^{-3}
$\alpha^{-1} = 137.0388 \pm 0.0006$	137.0352
$e = (4.80298 \pm 0.00006) \times \times 10^{-10}$ esu	4.80336×10^{-10} esu
$h = (6.62559 \pm 0.00015) \times \times 10^{-27}$ erg sec	6.62645×10^{-27} erg sec
$m = (9.10908 \pm 0.00013) \times \times 10^{-28}$ g	9.10979×10^{-28} g
$N* = (6022.52 \pm 0.09) \times \times 10^{20}$ mole^{-1}	6022.05×10^{20} mole^{-1}

* N is expressed on the unified scale of atomic weights on which $^{12}C = 12$.
** The standard deviations for the new values are omitted since the s. d. for the 26 ppm increase is undetermined.

Table 5. Alternative values of certain fundamental constants, depending on choice of value of α.

(resulting from these increases in α) between Spijkerman's result and the $N\Lambda^3$ data. Both Spijkerman's work and much of the $N\Lambda^3$ data were obtained in one and the same laboratory group at Johns Hopkins University. The apparent disagreement has been a puzzling unexplained riddle to this group, we have been told by them, and perhaps here they may have an explanation.

In Table 5 I list for comparison the numerical values of a few important fundamental constants obtained as the result of our Cohen–DuMond 1963 least-squares adjustment and as they will have to be modified if an increase of 26 ppm, relative to the value of α which we adopted in 1963, is found to be required assuming that no other changes or additions to our input data intervene.

These results illustrate clearly the highly interrelated nature of the fundamental constants. Dependent as our knowledge of them is upon many different fields of physics, we have here a good example of the importance of making occasional analyses of the consistency situations of sufficiently inclusive scope to serve as valuable guides to further research. The present example emphasizes especially that a better knowledge of the Sommerfeld constant, α, would be of great value to physics at the present time.

5. How May Our Present Dilemma Regarding α be Resolved?

What experimental methods offer promise at the present time of resolving the dilemma regarding the two discrepant values of α? The hf splitting in muonium of Hughes [5] can probably be refined and made more accurate when more intense sources of muons can be made available. The fs splitting in H or D by the new technique of Robiscoe [6] can be improved as to accuracy. Perhaps the most promising approach, in the opinion of Lamb, Jr., however, is the method of Crane and Wilkinson [23] in which

the electron magnetic moment anomaly, $\mu_e/\mu_0 - 1$, is measured directly. To compute α with the required precision from such a measurement, the quantum electro-dynamically derived theoretical formula for μ_e/μ_0 carried out to include the sixth order term (the term in α^3/π^3) is needed. Fortunately a promising start has just been made in the direction of estimating the coefficient of this term by Drell and Pagels [24]. Other less direct means of resolving the α-dilemma requiring highly precise measurements in the field of cryogenics are discussed in the subsection, 2.4 (13) entitled "Fluxoid Quantization" in reference 4.

[23] D. T. Wilkinson and H. R. Crane, Phys. Rev. 130, 852 [1963].

[24] S. D. Drell and H. R. Pagels, Phys. Rev. 140, B 397 [1965].

A Note on the Theory of Decaying Systems and the Computation of Decay Rates

H. Kümmel

Research Institute for Theoretical Physics, Helsinki *

Dedicated to Prof. J. Mattauch on his 70th birthday

The theory of decaying systems of Franz [1]—which seems finally to settle the basic questions left open so far—has been reformulated assuming a slightly altered initial situation. The currently used formulae for the decay rates (see Mang [2]) have been derived as an approximation to this practically rigorous theory.

The long standing problem of how to describe instable systems recently has found a surprisingly simple and probably final answer in a paper by Franz [1]. By carefully studying the actual situation at the beginning of the decay rate measurement, which implies that there must be a macroscopic time for the preparation of radioactive systems, he was able to avoid the drawbacks of the current theories: In the older formulations one has used complex energies and unnormalizable wave functions, not quite understanding how they come in. Although it has been clear for a long time that the poles of the S matrix (or the wave functions) are related to quasistable (resonant) states, it was not clear why these unphysical states had to be used. Also suitable initial wave packets — within reasonable limits — yield exponential decay laws. Others do not. Thus, it still remained the question to be settled why nature or man always seem to produce just the initial

situation which at the end leads to exponential decay. Also in Rosenfeld's recent brillant paper [3] this question has not been asked. Apparently, there must be a common reason yielding the same decay law, however man or nature has prepared the system. What is common to all those physical systems is that production and measurement are macroscopic manipulations, and thus it for instance is impossible to fix the time of the beginning of the measurement microscopically. To have seen this is the merit of Franz. Fortunately, the theory resulting from this idea is very simple and thus also in this respect is superior to other ways to attack this problem. It is gratifying that not only can one understand the cause for the exponential decay law, one also may search for reasons for deviations from it. Such deviations always occur at very large times. Also some of the conditions derived and used below may be violated. This may be of some relevance to the

[1] W. Franz, Z. Phys. 184, 181 [1965].

[2] H. J. Mang, Ann. Rev. Nucl. Sci. 14, 1 [1964].

[3] L. Rosenfeld, Nucl. Phys. 70, 1 [1965].